

Electric and Magnetic Fields

Chapter:-2

Electric Current: Electric current is defined as the rate of flow of negative charges of the conductor. Let, q be the charge flows through any cross section in time t , then electric current is, $I = \frac{q}{t}$

If small amount of charge dq flows in small time dt . So, electric current $I = \frac{dq}{dt}$

In S.I. system the unit of current is coulombs/second or ampere and its symbol is 'A'

i.e. $1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$

A current flowing through a conductor is said to be one ampere, if 1 coulomb of charge flows through the conductor in 1 second. Smaller unit of the current are mill ampere (mA) and micro ampere (μA)

Electrostatics: The branch of science which deals with phenomena due to attraction or repulsions of electric charges but not dependent upon their motion.

Electric Charge:

Electric Charge is defined as the amount of energy or electrons passing from one body to another either by conduction, induction or other specific methods. There are two types of electric charges namely positive charge and negative charges.

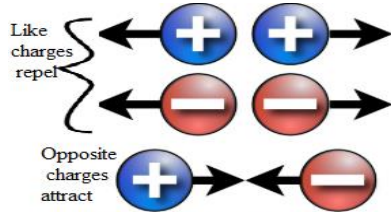
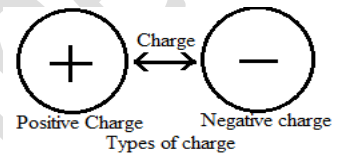


Fig: Basic nature of charges

The charge is denoted by the symbol 'q' and its standard unit is Coulomb. Mathematically charge is the number of electrons multiplied by the charge on 1 electron. i.e. $q = ne$

Where q is a charge, n is a number of electrons and e is a charge on 1 electron ($1.6 \times 10^{-19}\text{C}$)

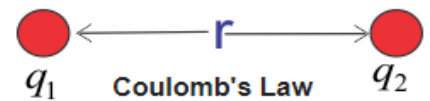
Unit of Charge: The S.I or Standard unit of electric charge is Coulomb. Its symbol is C and 1 C is defined as the charge flowing through a wire in 1 sec if the current flowing in the wire is 1 A. Charge of a body is measured by comparing it to a standard value.

Basic Properties of Electric Charge: As charges are of two types, positive and negative, there are other certain basic properties they follow. If the size of charged bodies is so small, we consider them as point charges. The basic properties of electric charges are as follows:

- a.** Charges are additive in nature
- b.** Charge is a conserved quantity
- c.** Quantization of charge

Forces between Electric Charges: The force of attraction or repulsion between two stationary point charges is directly proportional to the product of the magnitude of two charges and inversely proportional to the square of the distance between them. This force acts along the line joining the two charges.

Suppose, above figure consists of two point charges q_1 and q_2 charges. These two charges are separated by a distance r . Then according to Coulomb's Law the force F of attraction or repulsion between them is,



$F \propto q_1q_2$ (i) and $F \propto \frac{1}{r^2}$ (ii)

Combining eqⁿ (i) and (ii)

$F \propto \frac{q_1q_2}{r^2}$

or, $F = \frac{Kq_1q_2}{r^2}$ (iii)

Where, k is proportionality constant and its value depends on nature of medium separating q_1 and q_2 . The value of k depends on the nature of the medium between two charges and the system of units we choose to measure F, q_1 and q_2 and r. In SI units system, when two charges are in vacuum or air is,

$$k = \frac{1}{4\pi\epsilon_0} \quad \text{where, } \epsilon_0 \text{ is absolute permittivity of free space. Value of this constant in vacuum is } \epsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2} \text{ and } k = \frac{1}{4\pi\epsilon_0} = 8.998 \times 10^9 Nm^2 C^{-2} = 9 \times 10^9 Nm^2 C^{-2}$$

So, from above equation (1) the force between two charges located in air or vacuum, then the electrostatic force F is given by, $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

or, $F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$ In CGS system, K=1 then $F = \frac{q_1 q_2}{r^2}$

One Coulomb Charge:

We know that the electrostatic force between two charges in a vacuum is, $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2} \quad \text{If } q_1 = q_2 = q = 1 \text{ and } r = 1 \text{ then,}$$

$$F = 9 \times 10^9 \text{ N}$$

Hence 1C charge is defined as the amount of charge which would repel an equal similar charge placed at 1m away from it in vacuum or air with the force $F = 9 \times 10^9$.

i.e. 1 Coulomb = 3×10^9

$$1.6 \times 10^{-19} C = 1 \text{ electron}$$

$$1C = \frac{1}{1.6 \times 10^{-19}} \text{ electron}$$

$$1C = 6.25 \times 10^{18} \text{ electrons}$$

Relative Permittivity (Dielectric Constant):

Relative permittivity is the ratio of absolute permittivity of medium to the permittivity of vacuum. If ϵ is absolute permittivity of a medium and ϵ_0 is the permittivity of the vacuum, then relative permittivity of the medium is given by $\epsilon_r = K = \frac{\epsilon}{\epsilon_0}$

Suppose two charges q_1 and q_2 are placed at a distance r in a medium of permittivity ϵ and dielectric constant K and then in vacuum, the force between the charges in medium is given by,

$$F_m = \frac{q_1 q_2}{4\pi\epsilon r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 K r^2} \dots\dots(i) \text{ where } K = \frac{\epsilon}{\epsilon_0}$$

And for vacuum, $F_v = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \dots\dots\dots(ii)$

From eqⁿ (i) and (ii) $\frac{F_v}{F_m} = \frac{\frac{q_1 q_2}{4\pi\epsilon_0 r^2}}{\frac{q_1 q_2}{4\pi\epsilon_0 K r^2}} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \times \frac{4\pi\epsilon_0 K r^2}{q_1 q_2} = K$

Or, $K = \frac{F_v}{F_m}$

Thus, the **relative permittivity** of a medium can also be defined as the ratio of the electrostatic force between two charges placed at a distance in vacuum to that between the charges placed at the same distance in the medium.

For vacuum, $K = \frac{\epsilon}{\epsilon_0} = \frac{\epsilon_0}{\epsilon_0} = 1$ for any other dielectric media $K > 1$ i.e, for water $\epsilon_r = 8$, for perfect insulator $\epsilon_r = 0$, for perfect conductor $\epsilon_r = \infty$.

Forces between multiple electric charges:

It has been experimentally verified that force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to other charges are taken at a time. The individual forces remain unaffected due to other charges presence. This method is called the principle of superposition.

Let a number of charges $q_1, q_2, \dots, \dots, q_n$ are situated at different points in the space such that $r_{12}, r_{13}, \dots, \dots, r_{1n}$ be the distance of q_1 from $q_2, q_3, q_4, \dots, \dots, q_n$ respectively.

Total force $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}^2} \hat{r}_{13} + \dots, \dots, \dots + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_n}{r_{1n}^2} \hat{r}_{1n}$ Where $r_{12}, r_{13}, \dots, \dots, r_{1n}$ are the unit vector along $\vec{F}_{12}, \vec{F}_{13}, \dots, \dots, \vec{F}_{1n}$.

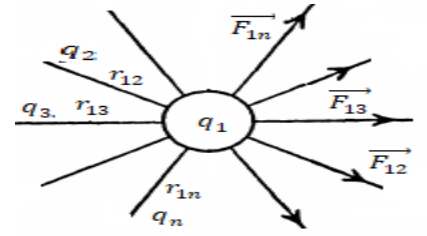


Fig: Force due to multiple electric charges

Electric Field:

The region around the electric charge in which the stress or electric force act is called an electric field or electrostatic field. If the magnitude of charge is large, then it may create a huge stress around the region. The electric field is represented by the symbol E. The SI unit of the electric field is newton per coulomb which is equal to volts per meter.

Electric Field Intensity:

The electrostatic force acting per unit positive charge on a point in electric field is called electric field intensity at that point. i.e. electric field intensity $\vec{E} = \frac{\vec{F}}{q_0}$. Its SI unit is NC^{-1} . It is a vector quantity and its direction is in the direction of electrostatic force acting on positive charge.

OR

Electric field intensity is the measure of intensity or strength of electrical force per unit charge at any given point in the electric field. It is denoted by the letter \vec{E} and its Unit is Newton per Coulomb (N/C). If F be the force experienced by a test charge q_0 placed at any point in an electrostatic field, then electric field intensity is given by;

$\vec{E} = \frac{\vec{F}}{q_0}$ here, we described the electric field intensity due to a point charge and several point charges;

1. Electric field intensity due to a point charge;

Suppose, a test charge q_0 is placed at a distance r from a point source charge q as shown in fig. If the system is in vacuum then, electrostatic force experienced by q_0 from q is given by

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2}$$

Therefore, from the definition, the electric field intensity \vec{E} of point charge q at a point where q_0 is placed

$$\text{by, } \vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \times \frac{1}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\text{Therefore, electric field intensity } \vec{E} = \frac{q}{4\pi\epsilon_0 r^2}$$

If the system is in medium of dielectric constant K then electric field intensity at same point from same source will be $\vec{E} = \frac{q}{4\pi\epsilon r^2}$ since $\epsilon = \text{permittivity of a median and } K = \frac{\epsilon}{\epsilon_0}$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0 r^2 K}$$

2. Electric field intensity due to several point charge:

Electric field intensity due to several point charges at a given point is measured in term of resultant field intensities of individual charges at that point and given by vector sum of intensities of individual charges.

i.e. $\vec{E}_1, \vec{E}_2, \dots, \dots, \vec{E}_n$ be the field intensities due to charges $q_1, q_2, \dots, \dots, q_n$ respectively at a point p at distance of $r_1, r_2, \dots, \dots, r_n$ respectively. The resultant electric field intensity at a point p is given by $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots, \dots, \dots + \vec{E}_n$

Suppose the system is vacuum $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} \hat{r}_1 + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2} \hat{r}_2 + \dots + \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_n^2} \hat{r}_n$

Where r_1, r_2, \dots, r_n are the unit vector along $\vec{E}_1, \vec{E}_2, \dots, \vec{E}_n$

If the system is in medium of dielectric constant $K = \frac{\epsilon}{\epsilon_0}$ then

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1^2} \hat{r}_1 + \frac{q_2}{r_2^2} \hat{r}_2 + \dots + \frac{q_n}{r_n^2} \hat{r}_n \right)$$

Electric Potential (V):

Electric potential at any points is equal to the work done per positive charge in carrying it from infinity to that point in electric field. i.e. electric potential $V = \frac{W}{q}$. It is a scalar quantity. Its SI unit is JC^{-1}

Electric Potential Difference:

A potential difference of one volt exists between two points when one Joule of work is required to move one Coulomb of charge from one point to the other. Between two points A and B we may write

$$\frac{W_{AB}}{q_0} = V_{AB} \quad \text{where } V_{AB} = V_B - V_A \text{ is the potential difference between A and B.}$$

Note that W_{AB} is the work done by the electric field in moving the charge. The work done by the "external agent" is $-W_{AB}$.

1. Electric Potential at a Point due to a Point Charge:

Suppose a point C at a distance r from charge +q where, electric potential is to be calculated. Let for an instant +1C charge is at a point A which is x distance from +q charge.

If system is in vacuum, electrostatic force \vec{F} acting on +1C charge is given by

$$\vec{F} = \frac{q}{4\pi\epsilon_0 x^2} \dots \dots \dots (i)$$

Therefore, small work done dw to bring +1C charge from point A to B against \vec{F} . Now, the total work done in bringing +1C charge from infinity to point C is given by,

$$dW = -Fdx \dots \dots \dots (ii)$$

$$W_{C\infty} = \int_{\infty}^C dW = - \int_{\infty}^r Fdx$$

$$W_{C\infty} = - \int_{\infty}^r \frac{q}{4\pi\epsilon_0 x^2} dx$$

$$W_{C\infty} = - \frac{q}{4\pi\epsilon_0} \int_{\infty}^r x^{-2} dx$$

$$W_{C\infty} = - \frac{q}{4\pi\epsilon_0} \left[\frac{x^{-2+1}}{-2+1} \right]_{\infty}^r$$

$$W_{C\infty} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^r$$

$$W_{C\infty} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{\infty} \right)$$

$$W_{C\infty} = \frac{q}{4\pi\epsilon_0 r}$$

$$\therefore W_{C\infty} = \frac{q}{4\pi\epsilon_0 r}$$

From the definition, this work done to bring +1C charge from infinity to point C is the electric potential of charge q at that point C.

$$\therefore W_{C\infty} = V_C = \frac{q}{4\pi\epsilon_0 r}$$

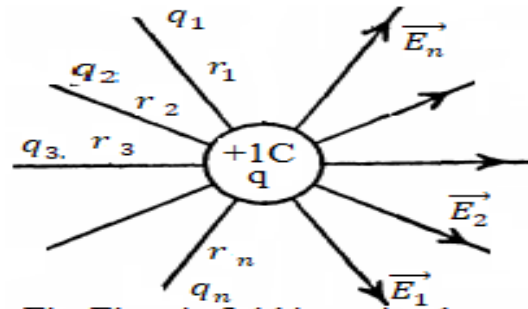


Fig: Electric field intensity due to several point charges

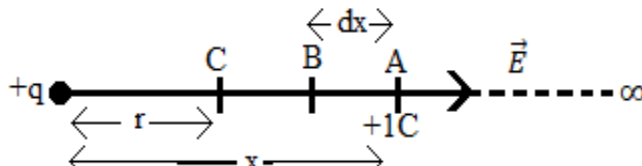


Fig: Calculation of electric potential at a point source charge

2. Electric Potential Difference between two Points Due to a Point Charge:

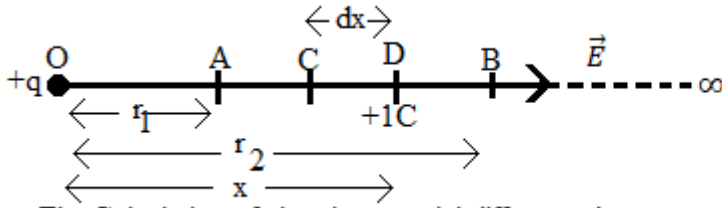


Fig. Calculation of electric potential difference between two points A and B due to a point charge

Suppose a point +q at a point O in vacuum whose electric field extends up to infinity. Let two points A and B in the electric field at distance r_1 and r_2 from O.

Suppose, +1 charge is at point D at a distance x from O. So, the electrostatic force experienced by it is $\vec{F} = \frac{q}{4\pi\epsilon_0 x^2}$ (i)

Now, the small work done dW to displace +1C charge from D to C against \vec{F} is given by $dW = -Fdx = -\frac{q}{4\pi\epsilon_0 x^2} dx$ (ii) here, (-) sign indicates that work is done against force \vec{F} , dx is the separation of point C and D.

Therefore, the total work done to displace +1C charge from B to A is given by,

$$W_{AB} = \int_B^A dW = -\int_{r_2}^{r_1} F dx$$

$$W_{AB} = -\int_{r_2}^{r_1} \frac{q}{4\pi\epsilon_0 x^2} dx$$

$$W_{AB} = -\frac{q}{4\pi\epsilon_0} \int_{r_2}^{r_1} \frac{1}{x^2} dx$$

$$W_{AB} = -\frac{q}{4\pi\epsilon_0} \int_{r_2}^{r_1} x^{-2} dx$$

$$W_{AB} = -\frac{q}{4\pi\epsilon_0} \left[\frac{x^{-2+1}}{-2+1} \right]_{r_2}^{r_1}$$

$$W_{AB} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{r_2}^{r_1}$$

$$\therefore W_{AB} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

From the definition, W_{AB} is equal to potential difference between points B and A due to point charge +q

i.e. $V_{AB} = V_A - V_B = W_{AB} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$

$$\therefore V_{AB} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Electrical (Electric) Potential Energy:

Electrical Potential Energy or Electrostatic Potential Energy as the energy that arises from the collection of charges when the charge will exert a force on any other charge. It is the total amount of work done in the bringing the charges from an infinite distance to their respective positions in the system.

Calculation of Electric Potential Energy:

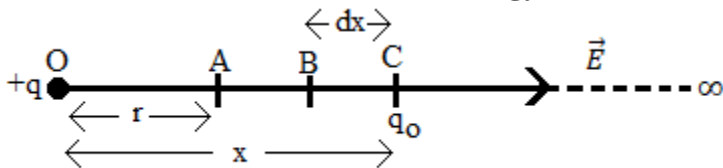


Fig. Calculation of electric potential energy

Let a point source charge +q at point O in vacuum and field of E extends up to infinity.

Another point A at a distance r from O, where electric potential energy is to be calculate. Here, another test charge +q₀ is at point C at distance x from O. therefore, +q₀ experiences repulsive electrostatic force which is given by $\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2} \dots\dots(i)$

Now small work done against \vec{F} to bring +q₀ charge from point C to B is given by

$$dW = -\vec{F} \cdot dx = -\frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2} dx$$

Here, (-) sign indicates that work is done against force \vec{F} , dx is the separation of point C and B. Therefore, the total work done to displace q₀ charge from ∞ to point A is given by,

$$W = \int_{\infty}^A dW = - \int_{\infty}^r \vec{F} \cdot dx = - \int_{\infty}^r \frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2} dx$$

$$W = -\frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^r \frac{1}{x^2} dx$$

$$W = -\frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^r x^{-2} dx$$

$$W = -\frac{qq_0}{4\pi\epsilon_0} \left[\frac{x^{-2+1}}{-2+1} \right]_{\infty}^r$$

$$W = \frac{qq_0}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^r$$

$$W = \frac{qq_0}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{\infty} \right)$$

$$\therefore W = \frac{qq_0}{4\pi\epsilon_0 r}$$

This work done is stored in the form of electric potential energy (U or W) at that point.

Magnet and Terms Related to Magnet:

- i. **Axis of Magnet**
- ii. **Equatorial Line**
- iii. **Geometric Length**
- iv. **Effective Length**
- v. **Magnetic Dipole**
- vi. **Magnetic Dipole Moment**
- vii. **Magnetic Meridian**
- viii. **Horizontal Component of Earth's Magnetic Field**

Coulomb's Law:

It states that the force of attraction or repulsion between any two poles is directly proportional to the product of their poles strength and is inversely proportional to the square of the distance between these two poles.

If m₁ and m₂ are the pole strength of two poles, then magnetic force between them F is,

$$F \propto m_1 m_2 \dots\dots\dots(i)$$

And if r be the distance between two poles then, $F \propto \frac{1}{r^2} \dots\dots\dots(ii)$

Combining eqⁿ(i) and eqⁿ(ii)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = K \frac{m_1 m_2}{r^2} \dots\dots\dots(iii) \text{ where } K \text{ is proportionality constant and its value depends on the system of unit chosen and medium.}$$

In CGS K=1, i.e. $F = \frac{m_1 m_2}{r^2} \dots\dots\dots(iv)$

In SI, $K = \frac{\mu_0}{4\pi}$ when two poles are placed in vacuum, then force between their becomes

$$F = \frac{\mu_0}{4\pi} \frac{m_1 m_2}{r^2} \dots\dots\dots(v)$$

Here, μ_0 be the permeability of free space and its value is $\mu_0 = 4\pi \times 10^{-7} \text{NA}^{-2}$ or WbA^{-1} or Hm^{-1} .

When two poles of magnets are placed in medium of permeability μ , then force between them becomes

$$F = \frac{\mu}{4\pi} \frac{m_1 m_2}{r^2} \dots \dots \text{(vi)} \quad \text{The dimensional formula of permeability } \mu \text{ is } [\text{MLT}^{-2}\text{A}^{-2}]$$

Magnetic Field: The space surrounding a magnet in which magnetic force can be experienced is called magnetic field. It has both magnitude as well as direction.

Magnetic Lines of Force:

To describe the phenomena related to magnets, lines are used to depict the force existing in the area surrounding the magnet. These lines are called the magnetic lines of force. These lines do not exist actually, but are imaginary lines that are used to illustrate and describe the pattern of the magnetic field

Magnetic Field Intensity:

The magnetic field intensity at any point is defined as the force experienced by the point when placed under the influence of an external magnetic field.

If F be the force between unit North Pole and North Pole of bar magnet, then magnetic field intensity B is defined as;

$$B = \frac{F}{m_N}$$

This is the magnetic field intensity at that point. The magnetic field intensity is a vector quantity and its direction is always from North Pole to South Pole of a magnet.

Force on Moving Charge in a Magnetic Field:

Consider, $+q$ electric charge is moving with velocity v through a magnetic field B . Then, magnetic force F_m exerted on the charge. The $+q$ charge is moving along XY plane making an angle θ with the direction of B , where B acting along Y -axis as shown in fig.

The electric charge $+q$ experiences a magnetic force F_m perpendicular to the plane of v and B . Then, it is found that the magnitude of magnetic force is

- i. Directly proportional to the magnitude of the charge i.e.

$$F_m \propto q$$

- ii. Directly proportional to the velocity of the charge i.e. $F_m \propto v$

- iii. Directly proportional to the strength of magnetic field i.e.

$$F_m \propto B$$

- iv. Directly proportional to sine of the angle between v and B

$$\text{i.e. } F_m \propto \sin\theta$$

$$F_m \propto qvB\sin\theta$$

$$\theta = 90^\circ \text{ Where } C \text{ is proportionality constant and its value is } 1.$$

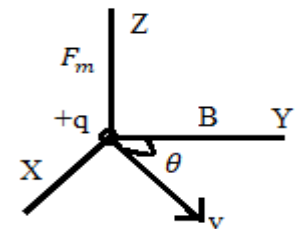


Fig: force on moving charge in a magnetic field

In vector form, $\vec{F}_m = q(\vec{v} \times \vec{B})$. This force is called Lorentz magnetic force.

Special Cases:

Case-I: if $\theta = 0^\circ$ or 180° i.e. $F_m = qvB\sin 0 = 0$

Case-II: if $\theta = 90^\circ$ i.e. $F_m = qvB\sin 90^\circ = qvB$

Case-III: if $v = 0$ i.e. $F_m = q \times 0 \times B\sin\theta = 0$

Case-IV: if $q=0$ i.e. $F_m = v \times 0 \times B\sin\theta = 0$

Unit and dimension of B: We have $F_m = qvB\sin\theta$ i.e. $B = \frac{F_m}{qv\sin\theta}$ when $q=1\text{C}$, $v=1\text{msec}^{-1}$, $\theta = 90^\circ$ and

$$F_m=1\text{N then, } B = \frac{1}{1 \times 1 \times 1} = 1 \text{ Tesla}$$

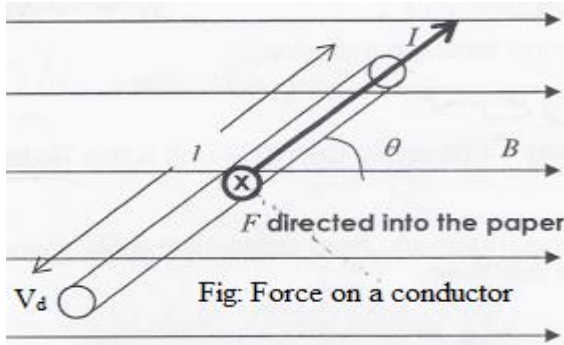
In SI system unit of magnetic field is tesla T. hence, magnetic field strength at a point is 1T if 1C charge moving with 1ms^{-1} at right angles to the magnetic field, experiences 1N force at the point. Again we have,

$$B = \frac{F_m}{qv\sin\theta} \quad \text{dimension of B} = [\text{MA}^{-1}\text{T}^{-2}]$$

Force on a Current Carrying Conductor in a Magnetic Field:

A conductor consists of large no of free electrons. Current on the conductor means drifting of such a free electron in any fixed direction due to the motion of such a free electron each electron experience magnetic force and hence conductor itself experience magnetic force. So, when a current carrying conductor is placed in a uniform magnetic field it experience force.

Let us consider a conductor having length l , cross-section area A in a uniform magnetic field. If n be the number of electrons per unit volume, V_d be the drift velocity of electron having electronic charge e then the current on the conductor.



Therefore, Force on each electron, $\vec{F}_e = e(\vec{V}_d \times \vec{B})$ (i)

Magnitude of each electron, $F_e = eV_d B \sin\theta$ (ii)

There are n free electron in the length l of a conductor. So, magnitude of total force F acting on the conductor is given by, $F = nA l F_e$ (iii) where $nA l =$ Number of electron in a volume.

i.e. $n = \frac{N}{Volume} = \frac{N}{Al}$

from eqⁿ(ii) and eqⁿ(iii)

$F = nA l e V_d B \sin\theta$

$F = n e V_d A l B \sin\theta$

$F = B I l \sin\theta$ (iv)

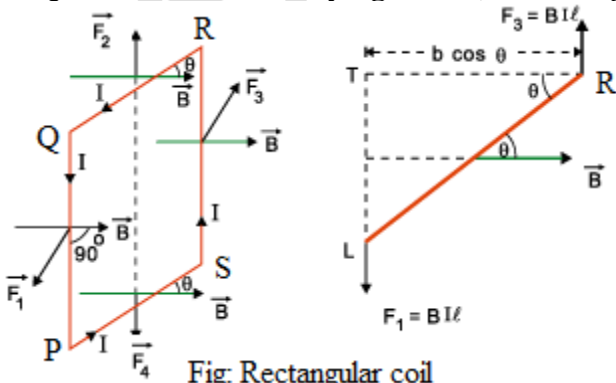
In vector form, $\vec{F} = I(\vec{l} \times \vec{B})$ is called Lorentz force. The direction of this force is perpendicular to the plane containing \vec{l} and \vec{B} . It can be found by using Fleming's left hand rule.

Some Cases:

Case-I: if $\theta = 0^\circ$ or 180° i.e. $F = B I l \sin 0^\circ = 0 =$ minimum i.e. current carrying conductor is placed parallel to the magnetic field, the conductor experiences no force.

Case-II: if $\theta = 90^\circ$ i.e. $F = B I l \sin 90^\circ = B I l =$ maximum i.e. current carrying conductor is placed at right angle to the magnetic field, the conductor experiences maximum force.

Torque on Current Carrying Wire (A Rectangular Coil) in a uniform Magnetic Field:



Let a rectangular coil PQRS be suspended in a uniform magnetic field \vec{B} . Consider $PQ = RS = l$ and $QR = SP = b$. If I be the current flowing in the coil in the direction SRQP and θ , the angle, which the plane of the coil makes with the magnetic field, then there will be forces acting on the four arms of coil. Let \vec{F}_i be the forces acting on the four current carrying arms PQ, QR, RS and SP of the coil (where $i = 1$ to 4, respectively).

The force on arm RQ is given by $\vec{F}_4 = I(\vec{RQ} \times \vec{B})$

And $\vec{F}_4 = I(RS)B \sin(180^\circ - \theta) = IbB \sin\theta$

The direction of this force is in the direction of $(\vec{RQ} \times \vec{B})$

The force on PS is i.e. $\vec{F}_2 = I(\vec{PS} \times \vec{B}) = IbB \sin\theta$

The direction of this force is in the plane of the coil directed downwards.

Since \vec{F}_2 and \vec{F}_4 are equal and opposite, they cancel each other.

Force on arm QP is $\vec{F}_1 = I(\vec{QP} \times \vec{B})$

$\vec{F}_1 = I(QP)B \sin 90^\circ = IlB$

From Fleming left-hand rule, the direction of force is perpendicular to the plane of the coil and is directed outwards.

Force on arm RS is, $\vec{F}_3 = I(\vec{RS} \times \vec{B})$

$\vec{F}_3 = I(RS) \sin 90^\circ = IlB$

Again, Fleming left-hand rule suggests that the direction of this force is perpendicular to the plane of the coil and directed away from the paper. Forces acting on QP and RS are equal, opposite and parallel constituting a couple. It produces rotation in the coil in anticlockwise direction.

The torque on the coil is $\tau = \text{Either force} \times \text{arm of the couple}$

Arm of couple = RT = RQ $\cos \theta = b \cos \theta$.

Therefore, $\tau = IlB \times b \cos \theta = IBA \cos \theta$

For a rectangular coil of n turns, $\tau = BInA \cos \theta$

Special Cases:

- If plane of the coil is parallel to the direction of magnetic field, then $\tau = BInA$
This is in the case of a radial field.
- If coil is set with its plane perpendicular to the direction of magnetic field B then $\tau = 0$

Magnetic Dipole Moment:

The magnetic dipole moment μ is defined as the product of magnetic current through the loop or wire and the area of the loop, which is given by $\mu = IA$ (i)

Where A is the area of the loop and I be the current flowing through the loop.

We know the definition of torque $\tau = BIA \sin \theta$ (ii)

Using eqⁿ(i) in eqⁿ(ii), we get $\tau = \mu B \sin \theta$ (iii)

In vector form, we can write eqⁿ(iii) as

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (iv) Since, $\vec{\mu}$ and \vec{B} are vector quantities

eqⁿ(iii) in eqⁿ(iv) gives the magnitude of the torque. The magnetic dipole experiences a torque when placed in and external field, work must be done to change its orientation and this work done is referred as energy of dipole.

The potential energy E_p of the dipole recalling that only changes in potential energy are experimentally observed, we must defined a zero or reference orientation. It is customary to set $E_p=0$ when $\theta = 90^\circ$ that is, when the dipole vector is perpendicular to the magnetic field.

To calculate E_p for any other orientation of μ , we calculate the work. Therefore, the term energy is given by $E_p = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} \mu B \sin\theta d\theta = \int_{90^\circ}^{\theta} \mu B \sin\theta d\theta = -\mu B \cos\theta$

\therefore the dipole energy, $E_p = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B} \dots\dots(v)$

The expression of E_p can be written as a dot product

$E_p = -\vec{\mu} \cdot \vec{B} \dots\dots(vi)$ this gives the energy of dipole.

Some cases:

Case-I: When $\theta = \pi$, $(E_p)_{max} = \mu B$, which is maximum dipole energy.

Case-II: When $\theta = 0$, $(E_p)_{min} = -\mu B$, which is minimum dipole energy.

Hall Effect:

When a magnetic field is applied perpendicular to current carrying conductor a voltage is developed across the specimen in a direction perpendicular to both the current and magnetic field. This phenomenon is called the Hall Effect. The voltage is so developed is called Hall voltage.

Let us consider, a rectangular type of specimen carrying current I_x in the X-direction and uniform magnetic field B_z is applied along the Z-axis which is perpendicular to I_x . The emf or voltage is developed along y-axis as shown in fig.

When the magnetic field is applied to the specimen, electrons are accumulated on the lower surface, producing net negative charge at there, simultaneously positive charges are accumulated on the upper surface. This combination of positive and negative surface charges create a downward electric field is called Hall field.

The Lorentz force F_L is produced which opposes by the field created by the surface charges produces a force. So that $F_H = F_L$

or, $eE_H = eV_x B_z$

$\therefore E_H = V_x B_z \dots\dots(1)$

The electric current density is given by, $J_x = n(-e)V_x \dots\dots(2)$

Here, n is the number of electrons which are taken as -ve charge. Dividing eqⁿ(1) and eqⁿ(2), we get

$$\frac{E_H}{J_x} = \frac{V_x B_z}{n(-e)V_x}$$

$\therefore \frac{E_H}{J_x B_z} = \frac{1}{n(-e)} \dots\dots(3)$

We know that, Hall field is proportional to both current density and magnetic field. So.

$E_H \propto J_x B_z$

$\frac{E_H}{J_x B_z} = Constant = R_H = Hall Constant \dots\dots(4)$

Using eqⁿ(3) and eqⁿ(4), we get ; $R_H = \frac{1}{n(-e)}$ in general $R_H = \frac{1}{nq}$, where q=charge.

The Hall constant or coefficient is used to find the nature of charge carrier. If $R_H =$ Positive, the charge carrier are holes i.e. $e=q$ but if $R_H =$ negative, the charge carrier are electrons i.e. $q=-e$.

Hence, Hall effect determines the nature of charge carriers.

Applications:

- i. Nature of charge carrier is known from the sign of R_H .
- ii. Concentration of carrier n can be evaluated
- iii. Mobility of carrier can be measured.

Electromagnetic Waves:

The waves which do not require medium for their propagation are called electromagnetic waves. These waves propagates in vacuum with speed of light $C = 3 \times 10^8 m/s$ and on account of being changeless. They are not deflected by electric and magnetic fields. There are 7 types of electromagnetic waves they are;

| Radiation | f (Hz) | λ (m) | Photon Energy (eV) | Comments |
|----------------------|--|--|--|---|
| power lines | 50 | 6×10^6 | 2×10^{-13} | Currents. Radiation emitted from 50 Hz power lines – are they dangerous? |
| radio AM FM TV | $< 10^9$ | > 0.3 | $< 4 \times 10^{-6}$ | Currents, electronic circuits. AM FM radio, TV. |
| microwaves | 10^9 3×10^{11} | 0.3 1×10^{-3} | 4×10^{-6} 1×10^{-3} | Magnetron. $\lambda \sim 10$ mm to 30 m can penetrate atmosphere – satellite communications, mobile phones, police radar, microwave cooking (polar water molecule), medical – diathermy. |
| infrared | 3×10^{11} 4×10^{14} | 1×10^{-3} 8×10^{-7} | 1×10^{-3} 2 | Molecular vibrations & rotations, all objects. Why do you get warm standing in the sun? Thermography, IR satellite images, detecting tumors & cancers. |
| light | 3.9×10^{14} 7.7×10^{14} | 7.8×10^{-7} 3.9×10^{-7} | 2 3 | Outer electrons, incandescent lamps, lasers, arcs. Eye sensitive to this narrow band. White light mixture of all colours of visible spectrum. TV (red blue green). If a uranium atom enlarged to a size of a pea, $\lambda_{red} \sim 20$ m long. Sunlight $\sim 10^{17}$ photons $\cdot s^{-1} \cdot m^{-2}$. Blue light photons sufficient energy to disrupt chemical bonds. |
| ultraviolet | 8×10^{14} 3×10^{16} | 4×10^{-7} 1×10^{-8} | 3 1×10^2 | Inner & outer electrons. Ionising (4 eV photons break C-C bonds). Skin cancers, tans the skin, vitamin D synthesis, damage to eyes. O_3 layer absorbs UV – protection layer. Can kill micro-organisms. |
| x-rays | 3×10^{16} 10^{20} | 1×10^{-8} 3×10^{-12} | 1×10^2 4×10^5 | Inner electrons, x-ray tubes. Diagnostic medical-rays. Computed tomography CT scan. Crystal & molecular structures. |
| γ -rays | $> 10^{18}$ | 3×10^{-10} ↓ | $> 4 \times 10^3$ | Nuclei, accelerators. Sterilisation, food preservation, cancer treatment, medical diagnosis, flaw detection. Very penetrating – can pass through large thickness of concrete. |

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